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ESTIMATION OF LOUDSPEAKER DRIVER PARAMETERS

Ivo Mateljan, Marjan Sikora

Faculty of Electrical Engineering, R.Boskovica 32, 21000 Split, Croatia (ivo.mateljan@fesb.hr)

Abstract: *Thiele and Small have established an analogous circuit for loudspeaker driver analysis on low frequencies. Most measuring systems extend that model with circuits, serially added to voice coil resistance, that simulate lossy voice coil inductor at higher frequencies. This paper discusses three types of models for lossy inductor: L2R, L3R and L2RK (Thorborg) model that uses semi-inductance element. The physical explanation of these models is given by using transformer that couples voice coil and magnet pole piece. Next, the paper shows use of linear and nonlinear least square error (LSE) minimization method for the estimation of lossy inductance and Thiele-Small parameters. In order to achieve the robust estimation, a semi-analytic solution for setting initial parameters and iterative implementation of LSE method is defined. Experimental work shows that L3R and L2RK models give better results than standard L2R model. L2RK model is most accurate, but for use in simulations with standard circuit elements the L3R model is preferable.*

Key words: Loudspeaker driver parameters, voice coil inductance estimation, least square error minimization

1. INTRODUCTION

The theory of linear analogous circuit model for electro-dynamic loudspeaker driver is well known, still there are lot of discussion on (1) which circuit parameters are important for simulation of loudspeaker response, and (2) which measurement and parameters estimation methods are acceptable for robust estimation. In this introduction we describe elements of driver analogous circuits with concentrated parameters. Then we present the parameter estimation methods that are implemented in the PC based measurement system [1].

Fig. 1 shows simple wideband analogous circuit of an electro-dynamic loudspeaker that is mounted in an infinite baffle. The voice coil, that has electrical resistance R_E is coupled to two circuits. First coupling circuit simulates the coupling of voice coil to magnetic pole piece, which also can have a copper short-circuited ring. It is an inductive coupling with input impedance denoted as Z_{LE}. Real part of this impedance shows influence of eddy currents to input impedance. Imaginary part also gives the contribution of voice coil inductance. We call this element a lossy inductor. Second coupling circuit shows mechanical coupling to the driver moving membrane that has mass M_{MS}, mechanical resistance R_{MS} and mechanical compliance C_{MS}. Mechanical coupling is caused by magnetic force that is proportional to the voice coil current and force factor Bl, where B is magnetic induction and l is voice coil length.

At low frequencies lossy inductor impedance Z_{LE} has no influence. Fig. 2 shows analogous circuit that is used for the estimation of the low-frequency input impedance.

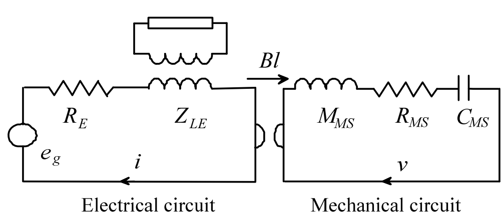

Fig. 1. Wideband analogous circuit of an electro-dynamic loudspeaker that is mounted in an infinite baffle

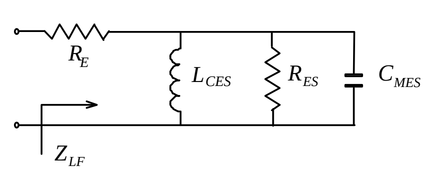

Fig. 2. Analogous circuit that is used for the estimation of low-frequency input impedance Z_{LF}

Elements of LF analogous circuit are: $L_{CES} = (Bl)^2 C_{MS}$, $R_{ES} = (Bl)^2 / R_{MS}$ and $C_{MES} = M_{MS} / (Bl)^2$.

Besides these physical driver parameters, Thiele and Small ([2], [3]) introduced set of parameters, called TSP, that easily characterize driver response as a second order high-pass filter. They are defined in Table 1.

Resonant frequency in free air (Hz)	$f_S = \dfrac{1}{2\pi\sqrt{M_{MS}C_{MS}}}$
Mechanical Q-factor	$Q_{MS} = \dfrac{2\pi\, f_S M_{MS}}{R_{MS}}$
Electrical Q-factor	$Q_{ES} = \dfrac{2\pi\, f_S M_{MS} R_E}{(Bl)^2}$
Total Q-factor	$Q_{TS} = \dfrac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}}$
Power available efficiency (%)	$\eta_0 = \dfrac{\rho_0}{2\pi c} \dfrac{S^2 (Bl)^2}{R_E {M_{MS}}^2}$
Sensitivity for 1W/1m, (dB re 20uPa)	$L_p(1W/1m) = 112{,}1 + 10\log(\eta_0)$
Equivalent acoustical volume (m^3)	$V_{AS} = \rho_0 c^2 S^2 C_{MS}$

Table 1. Thiele-Small parameters (TSP)

Thiele and Small also showed that loudspeaker parameters can easily be estimated from the measured magnitude of driver input impedance. The estimation procedure is given in the next section.

After estimation of TSP, other loudspeaker physical parameter can be estimated by Added mass method [3]. In that method, we first measure the impedance curve and estimate parameters f_S, Q_{MS} and Q_{ES}, for the loudspeaker that is mounted in the free air. Then we put an additional mass M_a on the membrane, measure impedance curve and estimate the shifted resonance frequency f_M and electrical Q-factor Q_{EM}. From equations for Q_{EM} and Q_{ES} we get:

$$M_{MS} = M_a \Big/ \left[\frac{f_S Q_{EM}}{f_M Q_{ES}} - 1 \right] \qquad (1)$$

When we know Q_{MS}, M_{MS} and f_S, it is easy to get the mechanical compliance C_{MS}, resistance R_{MS} and force factor Bl, by using equations that are defined in Table 1.

In the estimation procedures we use the value for voice coil resistance R_E, which is obtained by DC-ohmmeter measurement.

There are several different models for impedance of lossy inductor Z_{LE}. The simple theoretical model [4] expresses lossy inductor Z_{LE} as semi-inductance $K\sqrt{j\omega} = K\,(1+j)\sqrt{\omega/2}$, where K is constant expressed in unit semi-Henry (sH). The impedance of such semi-inductance increases with $\sqrt{\omega}$ rather than ω, to encounter effect of eddy currents in magnet pole piece. The similar model, as well as the procedure for estimation of its parameters, is given by Leach [5]. This pure semi-inductance model is not practical as engineers in many numerical simulations use some form of an analogous circuit that closely matches measurement data. The most commonly used circuit for the electrical voice coil impedance is serial connection of resistor R_E, inductor L_E and parallel connection of resistor R_2 and inductor L_2, as shown in Fig. 3a. It has been proven as useful model in many simulations. We call this model standard L2R model as it has been applied in most modern audio measurement systems ([1], [6] and [7]).

Besides this model, Fig. 3 also shows two models that better simulate impedance of lossy inductor: L3R and L2RK models. The L3R model [3] extends L2R model with one more parallel circuit L3||R3, while in L2RK Thorborg model [12] adds a semi inductance K parallel to L2||R2.

In section 3 it will be shown how to estimate parameters of these models.

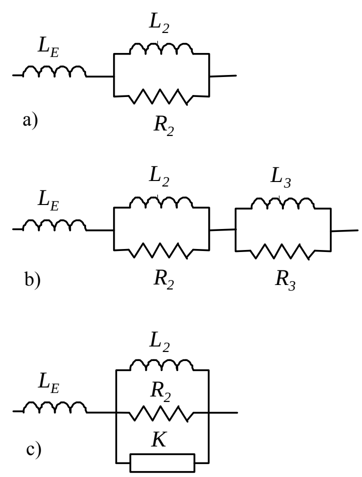

Fig. 3. Equivalent circuits for lossy inductance: (a) standard L2R model, (b) advanced L3R model and (c) L2RK model with semi-inductance K

2. ESTIMATION OF THIELE SMALL PARAMETERS

If we suppose that loudspeaker input impedance is measured accurately and without influence of noise, then loudspeaker parameters can be estimated by simple procedure given by Thiele and Small [2]. Here we present a slightly modified Thiele-Small procedure.

Fig. 4 shows typical curve of impedance magnitude vs. frequency. Denoted are: resonant frequency f_S, where impedance has maximum value Z_{max}, and two frequencies f_1, f_2, where impedance has the value $Z_{1,2}$. We use these values in the estimation procedure that is based on manipulation of the low frequency input impedance:

$$Z_{LF} = Z - Z_{LE} = R_E \frac{(1 - f^2 / f_S^2) + j\, f / (f_S Q_T)}{(1 - f^2 / f_S^2) + j\, f / (f_S Q_{MS})} \quad (2)$$

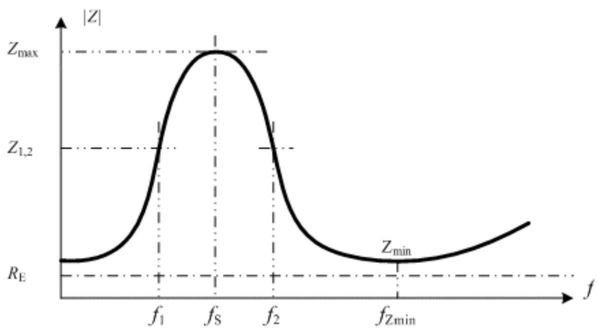

Fig. 4. Typical plot of impedance magnitude vs. frequency

At frequencies f_1 and f_2, (where $f_1 < f_S < f_2$, $f_1 f_2 = f_S^2$) impedance values are of equal magnitude $Z_{1,2} = r_1 R_E$. If we substitute this expression in the impedance equation, we get

$$\left|Z_{1,2}\right|^2 = r_1^{\,2} R_E^{\,2} = R_E^{\,2} \frac{r_0^{\,2} + \left(Q_{MS}\left(f_1 - f_2\right)/ f_S\right)^2}{1 + \left(Q_{MS}\left(f_1 - f_2\right)/ f_S\right)^2} \quad (3)$$

From this equation we get the mechanical Q-factor.
Now we can define a standard procedure for the measurement of Q-factors:

1. Measure a voice coil resistance R_E with a DC ohm-meter.
2. From impedance curve find f_S and Z_{max}.
3. Define $r_o = Z_{max}/R_E$.
4. Choose some impedance magnitude $R_E < |Z_1| < Z_{max}$ and find both frequencies (f_1 and f_2) where $Z = Z_1$.
5. Define $r_1 = Z_1/R_E$.
6. Calculate $Q_{MS} = \dfrac{f_s}{f_2 - f_1}\sqrt{\dfrac{r_0^{\,2} - r_1^{\,2}}{r_1^{\,2} - 1}}$ (4)
7. Calculate $Q_{ES} = Q_{MS} / (r_o - 1)$.
8. Calculate $Q_T = Q_{ES} Q_{MS} / (Q_{ES} + Q_{MS})$.

This procedure gives good results under two conditions:

1. if we have low noise level in measurement of impedance, and
2. if we first subtract magnitude of lossy inductor impedance $|Z_{LE}|$ from the magnitude curve.

In the next section it will be shown how to estimate the value of lossy inductor impedance. While it is unknown, in TSP estimation it is better to use values of Z_1 and f_1 from measured magnitude curve that are below the resonance frequency f_S and set $f_2 = f_s^2 / f_1$.

If we have noise in measurements we can reduce the estimation variance by repeating Thiele-Small procedure for different frequencies and average value of Q_{MS}. Good choices are frequencies where magnitude is 20% lower than maximum magnitude and 20% larger than minimum magnitude.
These two simple modifications of standard Thiele-Small procedure give more accurate and reliable estimation of TSP.
Thiele Small procedure is realizable only if the value of R_E is known, but sometimes that value is not known. In that case we can use the nonlinear least square error minimization procedure where we minimize squared difference between measured impedance Z_M and model impedance Z_{LF}, i.e. we define LSE function:

$$\varepsilon = \sum_f \left[\left|Z_M(f)\right| - \left|Z_{LE}(f)\right| - \left|Z_{LF}(f)\right|\right]^2 \quad (5)$$

where optimization variables are R_E, f_S, Q_{MS} and Q_T. $Z_{LE}(f)$ is semi-inductive part of lossy inductor impedance whose parameteras are also obtained by LSE minimization, as shown in the Section 3.
In the experimental work we used the Levenberg-Marquardt least-squares minimization procedures, as implemented in MINPACK optimization package [9]. An important step in the optimization is the setup of initial variable values. We use values obtained by Thiele-Small estimation procedure and $R_E = 0.9\, Z_{min}$.

Ex. 1	Thiele-Small R_E – fixed	LSE R_E - variable	Diff (%)
f_S (Hz)	45.28	45.80	1.15
R_E (Ω)	7.90	7.97	0.88
Q_{MS}	3.07	3.08	0.32
Q_T	0.27	0.27	0.00

Ex. 2	Thiele-Small R_E – fixed	LSE R_E - variable	Diff (%)
f_S (Hz)	33.93	34.00	0.20
R_E (Ω)	4.95	4.68	5.45
Q_{MS}	5.26	5.41	2.85
Q_T	0.32	0.31	3.13

Table 2. Two examples of Thiele-Small and LSE estimation. Diff (%) shows percentage difference.

Table 2 shows two examples of differences between parameters measured with Thiele-Small procedure and Levenberg-Marquardt least-squares error minimization.
In the first example differences are very small (in the order of 1%) but in the second example differences are much larger. After testing large number of driver we found that LSE minimization gives estimation of R_E with difference to DC value in worst case under 10%. A very important thing is that modified Thiele-Small estimation procedure gives results that are close to the LSE minimization results, if estimated R_E is close to the measured DC value of R_E. It means that extension of Thiele-Small two point method to many point estimation and averaging of Q, gives solution that is close to optimal one.
It is a common practice to measure TSP using large signal and small signal excitation. For example, in ARTA software for large signal excitation a stepped sine signal is used, while for small signal excitation a wideband noise signal is used. These two measurements give different values of impedance and estimated parameters, as driver significantly changes compliance and magnetic coupling under change from small signal to large signal excitation. This change influence resonant frequency, Q-factor and lossy inductor parameters. What is important to note is that in both cases the estimated parameters are correct, although they differ.

3. ESTIMATION OF VOICE COIL INDUCTIVE IMPEDANCE

For the estimation of parameters for three models of lossy inductor we developed two methods: (1) linear least-squares error minimization procedure for robust estimation of initial value for L2R model parameters and nonlinear Levenberg-Marquardt least-squares error minimization algorithm for final step of estimation procedure.
Next, the linear LSE minimization function will be defined by combining errors for real and imaginary part of Z_{LE}.

$$\mathrm{Re}(Z_{LE}) = R_2 \frac{\omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2} \tag{6}$$

$$\mathrm{Im}(Z_{LE}) = \omega L_1 + \frac{\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2} = \omega L_1 + \frac{R_2 \,\mathrm{Re}(Z_{LE})}{\omega L_2}$$

LSE function for imaginary part can be formulated as follows:

$$\varepsilon_1 = \sum_{i=1}^{n} \left(I_0(\omega_i) - \omega_i L_1 - \frac{R_2 R_0(\omega_i)}{\omega_i L_2}\right)^2 \tag{7}$$

where summation is for measured frequencies ω_i that are above frequency of impedance minimum. As a reference values we use measured impedance values corrected by low-frequency term:

$$I_0(\omega) = \mathrm{Im}(Z_M(\omega) - Z_{LF}(\omega))$$

$$R_0(\omega) = \mathrm{Re}(Z_M(\omega) - Z_{LF}(\omega)) \tag{8}$$

This way we get a linear LSE problem with two variables L_1 and ratio $y=R_2/L_2$. We obtain minimum of error when $\partial\varepsilon_1 / \partial L_1 = 0$ and $\partial\varepsilon_1 / \partial y = 0$. This condition results with matrix equation (9):

$$\begin{bmatrix} \sum_{i=1}^{n} \omega_i I_0(\omega_i) \\ \sum_{i=1}^{n} \frac{I_0(\omega_i) R_0(\omega_i)}{\omega_i} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \omega_i^2 & \sum_{i=1}^{n} R_0(\omega_i) \\ \sum_{i=1}^{n} R_0(\omega_i) & \sum_{i=1}^{n} \frac{R_0^2(\omega_i)}{\omega_i^2} \end{bmatrix} \begin{bmatrix} L_1 \\ y \end{bmatrix}$$

Solution of this equation gives value of L_1 and ratio $y=R_2/L_2$.
To get R_2, we define the error function for the real part of impedance:

$$\varepsilon_2 = \sum_{i=1}^{n} \left(R_0(\omega_i) - R_2 \frac{\omega_i^2 / y^2}{1 + \omega_i^2 / y^2}\right)^2 \tag{10}$$

where R_2 is a variable and ratio y is a constant. We minimize this error by setting $\partial\varepsilon_2 / \partial R_2 = 0$, which gives:

$$R_2 = \frac{\sum_{i=1}^{n} R_0(\omega_i) \frac{\omega_i^2 / y^2}{1 + \omega_i^2 / y^2}}{\sum_{i=1}^{n} \left(\frac{\omega_i^2 / y^2}{1 + \omega_i^2 / y^2}\right)^2} \tag{11}$$

$$L_2 = R_2 / y \tag{12}$$

What we have obtained is a noniterative solution for L2R circuit model parameters. This solution is suboptimal, as we have linearized LSE problem by algebraic manipulation, but it is robust and can serve as method for setting initial values for nonlinear LSE minimization.
Now we can formulate the nonlinear least square error minimization problem. For all models of Z_{LE} we want to minimize error function:

$$\varepsilon = \sum_{f} \left[\,|Z_M(f)| - |Z_{LF}(f)| - |Z_{LE}(f)|\,\right]^2 \tag{13}$$

where Z_M is measured impedance, Z_{LF} is a low-frequency impedance function calculated from initial TSP estimation, and Z_{LE} is lossy inductor impedance function. It is dependent on parameters L_1, L_2 and R_2 in L2R model, L_1, L_2, R_2, L_3 and R_3 in L3R model, and L_1, L_2, R_2 and K in L2RK model.
Initial parameters are obtained from linear LSE estimation of L2R model parameters. In L3R model we use the same L_1 value as in L2R model and use half of R_2 and L_2 value from L2R model as initial values for elements L_2, R_2, L_3

and R_3 in L3R model. In L2RK model we use K set to 0.2, L_2 is the same as in L2R model and R_2 is set to ten times the value of R_2 from L2R model.

These initial values give stable results with fast and convergent LSE iterations in all tested cases. Results will be illustrated for one loudspeaker, as results show the same behavior in all other tested cases.

As can be seen from Figures 5, 6, 7 and 8, none of lossy impedance models is ideal. L2RK model gives best results – only a small phase difference exists in the last octave of audio band. The L3R model also gives good results.

It is interesting to note the difference between Linear and Nonlinear LSE estimation of L2R model impedance (Fig. 5 and 6).

Linear LSE results show better fit of phase response while Nonlinear LSE results show better fit of magnitude response. It is expected behavior as Linear LSE minimization retains phase relationship (7), while Nonlinear LSE minimization uses only magnitude values.

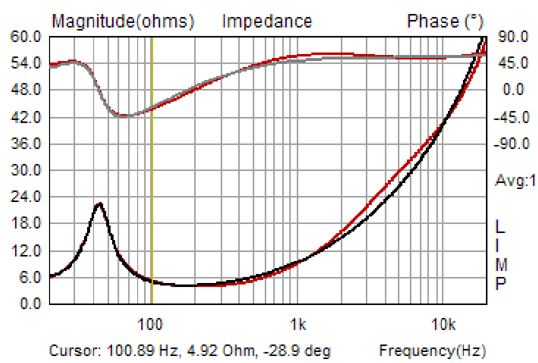

Fig. 5. Comparison of impedances obtained by measurement (magnnitude-black/phase-gray) and Linear LSE estimated L2R model (red)

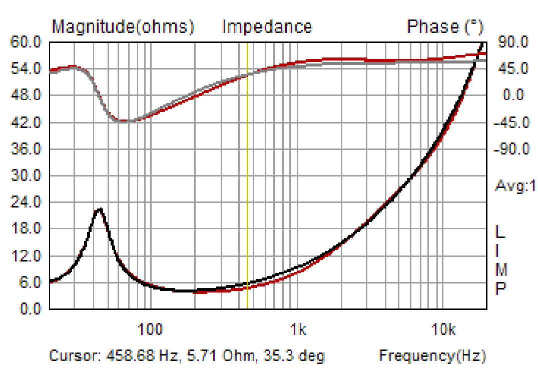

Fig. 6. Comparison of impedances obtained by measurement (magnitude-black/phase-gray) and Nonlinear LSE estimated L2R model (red)

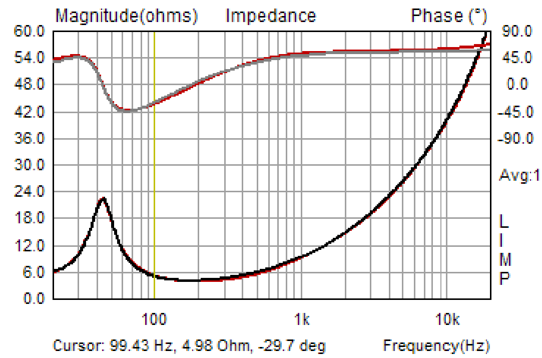

Fig. 7. Comparison of impedances obtained by measurement (magnitude-black/phase-gray) and Nonlinear LSE estimated L3R model (red)

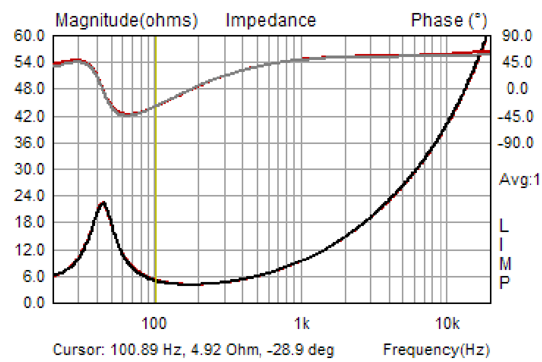

Fig. 8. Comparison of impedances obtained by measurement (magnitude-black/phase-gray) and nonlinear LSE estimated L2RK model (red)

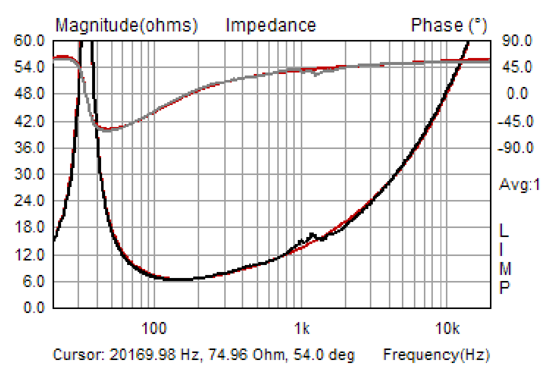

Fig. 9. Comparison of impedances obtained by measurement (magnitude-black/phase-gray) and estimated by L2RK model (red), for loudspeaker with resonances

Fig. 9 shows comparison of impedances obtained by measurement and by estimation using L2RK model, for the loudspeaker that exhibits resonances. We see that

resonant deviation does not degrade the quality of the estimation.
In all examples the fit of measured and estimated impedance curves is very good on low frequencies, where we used the modified Thiele-Small method for the estimation of loudspeaker parameters.

4. CONCLUSION

The main goal was to develop methods that will give robust estimation of loudspeaker parameters and loudspeaker analogous circuit elements. It was shown that by simple extension of Thiele-Small two point method for loudspeaker parameters estimation it is possible to obtain results that are in close agreement with nonlinear LSE methods, yet estimation procedure is numerically stable and gives good results in most cases. Nonlinear LSE minimization in some cases does not give good results, and measurement software must implement visualization component for the comparison of measured and estimated impedance curves.
The nonlinear LSE estimation must be used if we don't have the measured value of voice coil DC resistance.
The estimation results depend on measurement S/N. Results also depend on quality of measured driver. Some drivers exhibit large number of membrane/basket resonances and/or highly nonlinear behavior. In that case results of estimation procedure will be degraded.
For the estimation of lossy inductor impedance, three models are defined. Best results are obtained with L2RK Thorborg model that contains semi-inductance element. That model has arisen from the examination of voice coil magnetic coupling. It has clear physical explanation, but it is not practical for use in standard circuit analysis software. In that case, slightly less accurate L3R model, or standard L2R model are preferable.
Presented estimation procedures are combination of linear and nonlinear LSE minimization, as well as simple curve fitting methods. During the experimental work we tested large number of drivers to approve these methods as robust estimation techniques. The paper shows when to use each of these methods.
The estimation of loudspeaker driver parameters is implemented in many audio measurement systems, but very often they do not give the same results in the estimation of loudspeaker parameters. We think that it is desirable feature that all measurement systems give a full explanation of measurement procedures, excitation signals and estimation methods. Methods presented in this paper are implemented in the program LIMP that is the part of the ARTA software.

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