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MODELS FOR THE ESTIMATION OF THE LOUDSPEAKER IN-ROOM RESPONSE

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SUMMARY

Low-frequency and high-frequency models for the estimation of the loudspeaker in-room response are presented. A new method of acoustical images for directional sources has been developed. The theory holds for point sources with arbitrary directional characteristics, placed near reflecting perpendicular walls. Examples of frequency and impulse response for closed box loudspeaker, planar dipole loudspeaker and nondirectional acoustic source are presented.

1. INTRODUCTION

Well-known studies in the audio field for the estimation of a loudspeaker in-room power response were presented by Allison [4] and Ballagh [5]. They used the method of images, approximating loudspeaker as a point source, to calculate the power response of a loudspeaker which is mounted in the corner of three perpendicular walls. Adams [6] used the same method to estimate the point source power response in rectangular rooms. He stated that the power curve for steady-state conditions retains a trend in shape which follows the curve for the early sound wave. The practical consequence of this conclusion is that it is enough to use the three-wall model for the estimation of the power response of a loudspeaker that is located near the corner of a large rectangular room. Besides these simple methods, the modal wave function expansion [1,2] can be used as more accurate method. In the first part of this work the acoustic power response of point source, calculated by the method of images, is compared with a response that is calculated using the modal wave function series.

These methods fail to estimate the response of real loudspeakers at higher frequencies, where loudspeakers exhibit directional characteristics. By approximating the loudspeaker box with a spherical baffle it has been shown that even small loudspeaker boxes exhibit directional characteristics on frequencies above 100 Hz.

In the second part of this work, a new method for the estimation of frequency and impulse response of directional acoustical sources is derived. Real loudspeaker boxes are modeled as directional point sources. Then, the method of images is used to estimate the frequency response of directional sources that are placed in the corner of three perpendicular walls. Finally, the procedure for the loudspeaker time response estimation is proposed as a

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numerical calculation of the impulse response envelope from the single-sided frequency response data.

2. THE LOUDSPEAKER POWER RESPONSE - LOW-FREQUENCY MODEL

All known analytical and numerical methods for analyzing loudspeaker in-room acoustic power response treat the loudspeaker as a nondirectional point source. Loudspeakers exhibit nondirectional characteristics only at low frequencies, thus modeling the loudspeaker as a point source represents a low-frequency model for the loudspeaker response estimation.
The acoustic power of a nondirectional point source can be obtained from (see Appendix):

$$P_A = \mathrm{Im}\left(j\underline{U}\,\underline{p}^*(\mathbf{r}\rightarrow\mathbf{r_0})\right) = \mathrm{Re}\left(\underline{U}^*\,\underline{p}(\mathbf{r}\rightarrow\mathbf{r_0})\right) \qquad (1)$$

where U is the source volume velocity, $p(\mathbf{r})$ is the sound pressure at point $\mathbf{r}$ in bounded or unbounded space, $\mathbf{r_0}$ denotes location of a point source and * denotes complex-conjugate quantities. Underlining denotes complex steady-state variables.
If we divide Eq. (1) with the value of the acoustic power of the point source radiating in a free space ($P_{4\pi}=\rho_0 ck^2|U|^2/4\pi$) and substitute the value of acoustic radiation resistance in a free space ($R_{a4\pi}=\rho_0 ck^2/4\pi$), we obtain:

$$\frac{P_A}{P_{4\pi}} = \frac{1}{R_{a4\pi}}\mathrm{Re}\left(\frac{\underline{p}(\mathbf{r}\rightarrow\mathbf{r_0})}{\underline{U}}\right) = \frac{R_{ar}}{R_{a4\pi}} \qquad (2)$$

where R_{ar} is the point source acoustic radiation resistance.
Using Eq. (2), and the known analytical solution for the sound pressure field, we can calculate the acoustic power and radiation resistance of the point source in any bounded space.

The most widely used method for the estimation of the acoustic power in a large rectangular room is the method of images. In this method it is assumed that the sound field is dominantly affected only by reflections from walls which are closest to the source. If the loudspeaker is placed near the corner of a rectangular room, we can apply the method of images concerning only three perpendicular walls that form the corner; that is, we add seven image sources to satisfy the boundary value problem (Fig. 1).

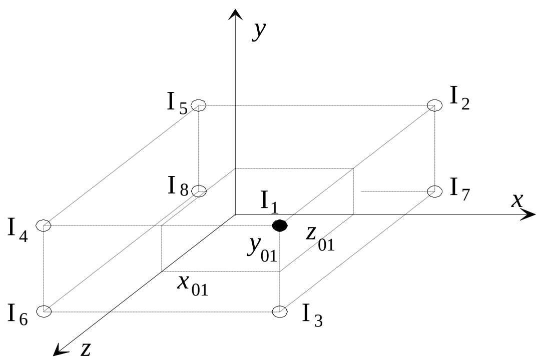

Fig. 1 The point source (I1) and seven images (I2..I8) at the corner of three perpendicular, rigid walls.

Assuming that the room corner coincides with a coordinate origin and that point source, with volume velocity U, is placed at point $\mathbf{r_{01}}$ (x_{01},y_{01},z_{01}), the total sound pressure at point $\mathbf{r}$ (x>0,y>0,z>0) is given by :

$$\underline{p_{uk}}(\mathbf{r}) = j\omega\rho_0 \underline{U} \sum_{i=1}^{8} \frac{e^{-jk|\mathbf{r}-\mathbf{r_{0i}}|}}{4\pi|\mathbf{r}-\mathbf{r_{0i}}|} \tag{3}$$

where $\mathbf{r_{0i}}$ (x_{0i},y_{0i},z_{0i}, i=2,3,..8) is vector distance of image sources from the origin. To obtain the acoustic power, we substitute Eq. (3) into Eq. (2) for the limiting case $\mathbf{r} \rightarrow \mathbf{r_{01}}$. We obtain:

$$\frac{P_A}{P_{4\pi}} = \frac{R_{ar}}{R_{a4\pi}} = \sum_{i=0}^{1}\sum_{m=0}^{1}\sum_{n=0}^{1} \frac{\sin\left(2k\sqrt{ix_{01}^{\ 2} + my_{01}^{\ 2} + nz_{01}^{\ 2}}\right)}{2k\sqrt{ix_{01}^{\ 2} + my_{01}^{\ 2} + nz_{01}^{\ 2}}} \tag{4}$$

Fig. 2 shows the frequency characteristic $P_A/P_{4\pi}$ for three source positions. The power enhancement (9dB) can be achieved only at very low frequencies. At higher frequency the radiation characteristic approaches free space condition (±2dB). The practical consequence of this result is that we can use Eq. (2) to quickly estimate possible loudspeaker positions for which we can expect the acceptable low-frequency response.

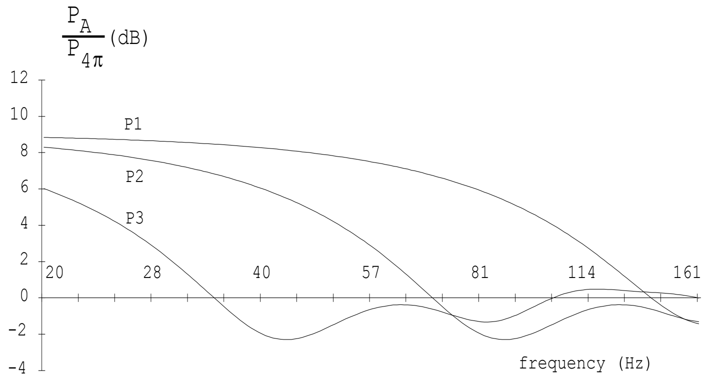

Fig. 2 The frequency characteristic of a point source acoustic power (normalized with a free-space acoustic power: $P_A/P_{4\pi}$). The source is located in the corner of three perpendicular walls: P1(0.6m, 0.75m, 0.3m), P2(1.5m, 1.0m, 0,8m) and P3(3.0m, 2.0m, 1.6m).

Further, we want to test the applicability of this method for the estimation of the acoustic power in smaller rooms.

To estimate the source acoustic power in small rooms (where wavelength approaches dimensions of the room), the calculation of all acoustic field resonant modes is necessary. This can easily be done if the source is placed in a rectangular room.

In a rectangular room (dimensions: L_x,L_y,L_z, volume V and wall surface S) with nearly rigid walls (specific acoustic impedance Z) the sound pressure at point $\mathbf{r}$(x,y,z), excited by a point source located at point $\mathbf{r_0}$(x_0,y_0,z_0), is given by [1]:

$$\underline{p} = -j\omega\rho_0 \frac{\underline{U}}{V} \sum_n \frac{\mathrm{F}(\mathbf{r},n)\mathrm{F}(\mathbf{r_0},n)}{k^2 - k_n^{\ 2} - jkB_n} \tag{5}$$

where: n=(n_x,n_y,n_z)=0,1,2,..,

$$k_n^{\,2} = \pi^2 \left[(n_x / L_x)^2 + (n_y / L_y)^2 + (n_z / L_z)^2 \right] \tag{6}$$

$$\mathrm{F}(\mathbf{r}, n) = a_x a_y a_z \cos\frac{n_x \pi x}{L_x} \cos\frac{n_y \pi y}{L_y} \cos\frac{n_z \pi z}{L_z} \tag{7}$$

$$a_x^{\,2}, a_y^{\,2}, a_z^{\,2} = (2 \ \text{ for } n > 0) = (1 \ \text{ for } n = 0), \tag{8}$$

$$B_n = \frac{1}{V} \iint_S \frac{\rho_0 c}{Z} \mathrm{F}^2(\mathbf{r}, n) dS \tag{9}$$

These equations hold for locally reactive room surfaces. Using Eq. (1) and (5) we obtain the source acoustic power, given by:

$$P_A = P_{4\pi} \frac{4\pi}{V} \sum_n \frac{B_n \mathrm{F}^2(\mathbf{r_0}, n)}{\left(k^2 - k_n^{\,2}\right)^2 + k^2 B_n^{\,2}} \tag{10}$$

If all wall surfaces have the same impedance Z, then

$$B_n = \frac{2\rho_0 c}{ZV} \left(a_z^{\,2} L_x L_y + a_y^{\,2} L_x L_z + a_x^{\,2} L_y L_z \right) \tag{11}$$

B_n is the complex value. The imaginary part of B_n can be neglected (this causes minor changes in the modes resonant frequency). The real part of B_n can be calculated from the known room reverberation time T_{60}, as for all higher modes (n>0) [1]:

$$\mathrm{Re}(B_n) = \mathrm{Re}\left(\frac{2\rho_0 cS}{ZV} \right) = \frac{6 \ln 10}{cT_{60}}. \tag{12}$$

(On very low frequencies, were only tangential or axial modes are excited (n=0), the reverberation time is higher then predicted by Eq. (12)).

To demonstrate this method, the acoustic power estimation is done for two rooms with T_{60}=1s; *Room1* represents typical domestic room (according to IEC-u: 6.7x4.1x2.8m), and *Room2* represents bigger room (11x9x4m). On figures 3. and 4. the acoustic power frequency characteristic ($P_A/P_{4\pi}$), 1/3 octave smoothed, is shown for a two source position. Also, at same figures, the acoustic power estimation obtained by the method of images is shown (this curve represents response in very large room).

At higher frequencies the acoustic power level is equal in all rooms and for all source positions. Since the response curve calculated by the method of images is close to response curves calculated by accounting room modes, we conclude that the loudspeaker response is mainly affected by reflections from walls which are closest to the loudspeaker. If a source position is near the corner of the room, the acoustic power level changes on frequencies below 150 Hz. After moving the source approximately 1m from the corner, the acoustic power level varies only ±2dB on frequencies above 50Hz. It is important to note that the response calculated using the method of images closely approximates in-room response only at higher frequencies. As it represents some kind of a mean response, it can be used as a first approximation in a acoustic power response estimation. In cases when we don't need to analyze the sub-bass response it is enough to use the method of images on three walls.

Fig. 3 The frequency characteristic of a point source acoustic power ($P_A/P_{4\pi}$) in three rooms: Room1(6.7x4.1x2.8m), Room2(11x9x4m) and a very large room. Source position: x=0.6m, y=0.75m, z=0.3m.

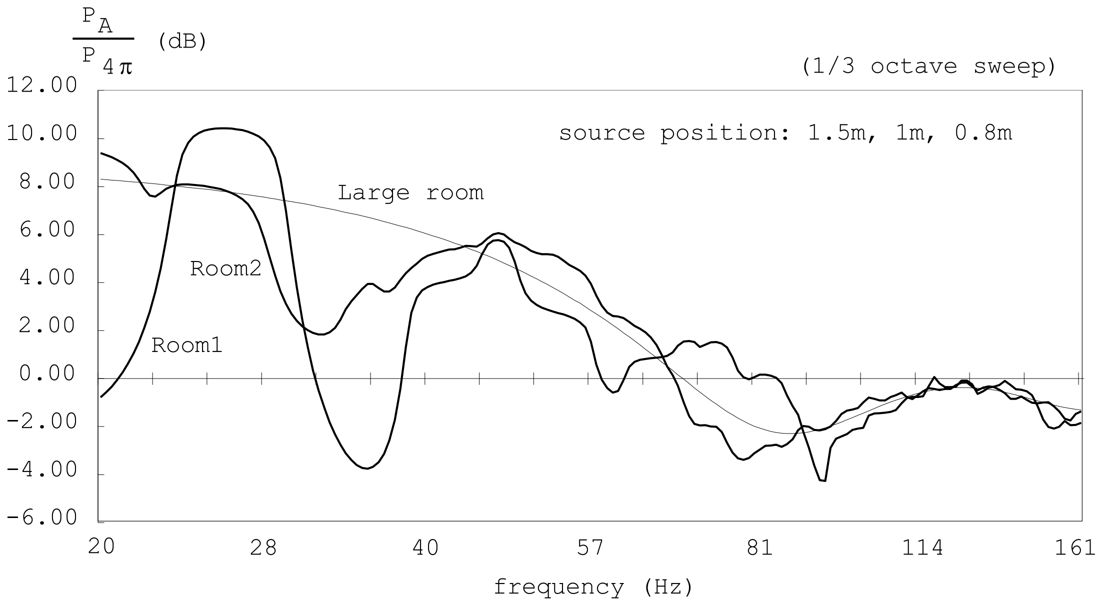

Fig. 4 The frequency characteristic of a point source acoustic power ($P_A/P_{4\pi}$) *in three rooms: Room1*(6.7x4.1x2.8m), *Room2*(11x9x4m), *and a very large room. Source position: x*=1.5m, *y*=1m, *z*=0.8m.

To prove the usefulness of this model we need to approve that loudspeaker boxes behave as nondirectional sources at frequencies below approximately 150 Hz.

To do this, we shall compare the pressure response of a loudspeaker which is mounted in an infinite baffle ($p_{2\pi}$) with a response of a loudspeaker which is mounted in a spherical box of diameter d ($p_{4\pi}$). Both cases are shown in Fig. 5.

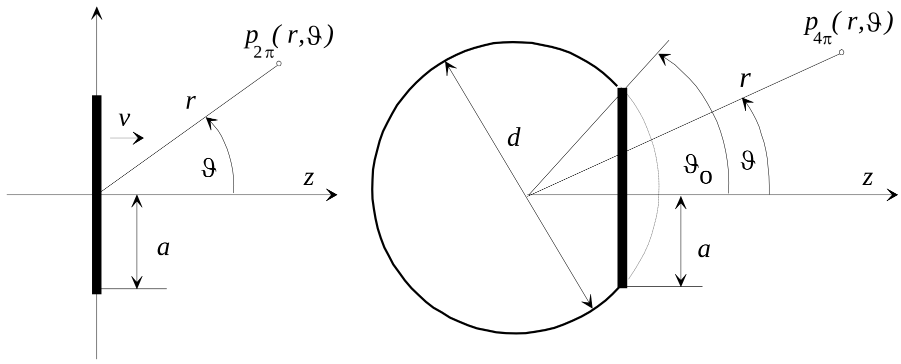

Fig. 5 Loudspeaker mounted in an infinite baffle and in a spherical box .

At low frequencies (*ka*<<1), the sound pressure at distance *r*, on the axis of a loudspeaker which is mounted in an infinite baffle, is given by:

$$p_{2\pi}(r) = j\frac{R_{ar2\pi}}{kr}Ue^{-jkr}, \tag{13}$$

where $R_{ar2\pi}=\rho_0 ck^2/2\pi$ is the radiation resistance in the half-space, $U = a^2\pi v$ (*a* - membrane radius, *v* - membrane velocity)

The sound pressure low-frequency response of the loudspeaker, which is mounted in a spherical box, can be calculated by the method of separable variables if we assume that the membrane has a spherical form [3] (shown dashed on Fig. 5). Then we obtain:

$$p_{4\pi}(r,\vartheta) = \rho_0 c\frac{e^{jkr}}{kr}\frac{v}{2}\sum_{n=0}^{\infty}C_n P_n(\cos\vartheta)\frac{e^{jn\pi/2}}{h_n^{(2)'}(ka)}, \tag{14}$$
$$h_n^{(2)'}(ka) = -jB_n(ka)e^{-j\delta_n(ka)}$$

$$C_n = \frac{n+1}{2n+3}\left(P_n(\mu) - P_{n+2}(\mu)\right) + \frac{n}{2n+1}\left(P_{n-2}(\mu) - P_n(\mu)\right), \qquad (\mu = \cos\vartheta) \tag{15}$$

where P_n are Legendre polynomials, $h^{(2)'}$ is a spherical Bessel function. We calculate $h^{(2)'}$ from tabulated data for B_n and δ_n ([3] and [1]).

If we use this result to get the normalized response ($p_{4\pi}/p_{2\pi}$) on the loudspeaker axis (ϑ =0), corrected for a phase delay $e^{-jkd/2}$, we obtain:

$$\frac{p_{4\pi}}{p_{2\pi}} = \frac{e^{-jkd/2}}{\left(\frac{kd}{2}\sin\vartheta_0\right)^2}\sum_{n=0}^{\infty}C_n\frac{e^{j(\delta_n + n\pi/2)}}{B_n(ka)} \tag{16}$$

The numerical calculation of this equation is quite complex for practical use, as it needs some table driven software (to interpolate B_n and δ_n values). An approximate form of this equation can be obtained by numerically fitting this equation to the transfer function form. For practical purposes we are interested in cases when the loudspeaker membrane occupies 1/40 to 1/10 of the spherical box surface ($5^{o} < \vartheta_0 < 20^{o}$). Then, Eq. (4) can be approximated, with the error less then ±0,3dB and ±3^{o}, by the frequency response function:

$$\frac{p_{4\pi}}{p_{2\pi}} = \frac{1}{2}\frac{1+j\omega\pi\frac{d}{2c}}{1+j\omega\pi\frac{d}{4c}} \tag{17}$$

This equation is equivalent to a transfer function with one pole and one zero. The reciprocal of this function can be used as a 2π/4π bass equalizer transfer function. It is a function of the box diameter and not a function of the membrane radius. Fig. 6 shows the frequency characteristic of this function for three boxes (*d*=1.0, 0.5 and 0.3m). It is important to note that only boxes with a diameter less than 0.3m exhibit non-directional characteristics at frequencies below 100 Hz. That's why we can conclude that methods in which the loudspeaker box is modeled as a nondirectional source, cannot give us an accurate estimation of the loudspeaker response.

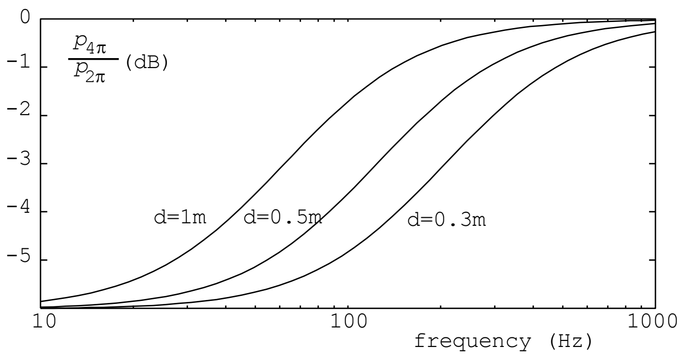

Fig. 6 The frequency characteristic ($p_{4\pi}/p_{2\pi}$) *of the sound pressure response on the axis of a loudspeaker which is mounted in a spherical box (with diameter:* 1, 0.5 *and* 0.3 *m), normalized with the response of a loudspeaker which is mounted in an infinite baffle.*

3. DIRECTIONAL LOUDSPEAKERS RESPONSE ESTIMATION

The goal is to establish a simple method for the estimation of a directional loudspeaker in-room response. As it has been shown in the preceding chapter, the loudspeaker in-room response is dominantly affected by the reflection from walls which are closest to the loudspeaker. So, if we analyze the response of the loudspeaker which is placed near the corner of the room, it is a good approximation to take into account only reflections from three walls that form the corner of the room. This approach is also correct from the psycho-acoustic standpoint, since early reflections (those in 20ms time window) have a much higher perceptual significance than late reflections.

To estimate the loudspeaker in-room response, we shall use the method of images on three perpendicular walls, but with a directional source characteristic included.

First, we approximate the loudspeaker box as a point directional source, that is, at some point ($x,y,z \Leftrightarrow r,\varphi,\vartheta$) in an unbounded space, the sound pressure is given by:

$$p(x,y,z,\omega) = p(r,\varphi,\vartheta,\omega) = W(j\omega)f(\varphi,\vartheta,j\omega)\frac{e^{-jkr}}{r} \tag{18}$$

where $W(j\omega)$ is the loudspeaker frequency response function and $f(\varphi,\vartheta,j\omega)$ is the directivity function (loudspeaker directional characteristic). In this approximation we discard the effect of field perturbation due to finite loudspeaker box size, and the influence of wall reflections as reactive forces on the loudspeaker membrane.

To adopt the method of images for directional sources, we assume that room corner coincide with a origin of a global coordinate system (x,y,z). The loudspeaker position is at point (x_{0i},y_{0i},z_{0i}) that is also the origin of a local coordinate system (x_i,y_i,z_i) (Fig. 7).

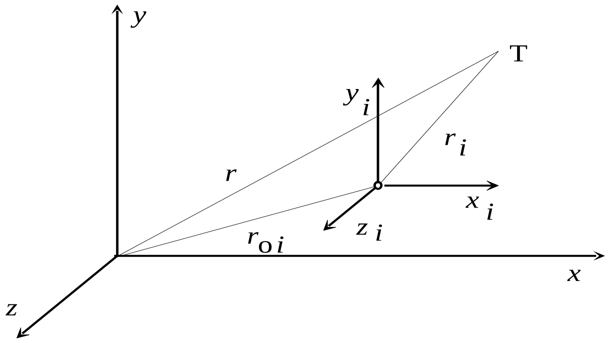

Fig. 7 Global and local coordinate system

Local coordinates are parallel to global coordinates, but unit vectors can be of different directions, that is

$$\mathbf{e_{xi}} = q_i \mathbf{e_x}, \quad \mathbf{e_{yi}} = u_i \mathbf{e_y}, \quad \mathbf{e_{zi}} = w_i \mathbf{e_z}, \qquad q_i, u_i, w_i = \pm 1, \quad i = 1,2,...8 \tag{19}$$

where q_i, u_i,and w_i are direction factors with two possible values: 1 or -1. Now, we can express the position of a point in a local coordinate system as a product of direction factors and coordinates of a global coordinate system, that is:

$$\mathrm{T}(x_i, y_i, z_i) = \mathrm{T}(q_i(x - x_{0i}), u_i(y - y_{0i}), w_i(z - z_{0i})) \tag{20}$$

This way, we can define eight different local coordinate systems. If in each of these coordinate systems we use the same expression for the acoustic pressure (Eq. 18), we obtain eight different directional characteristics in a global coordinate system.

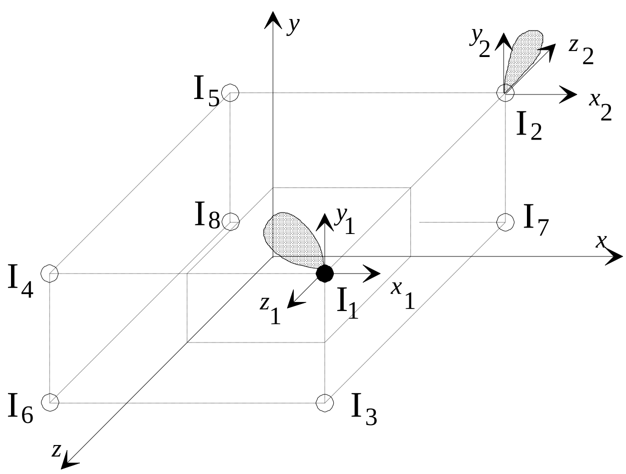

Fig. 8 Directional source (I_1) *and directional image sources* (I_2..I_8)

It is important to note that changing the sign of one direction factor, causes the direction change of one coordinate axis. This way, we obtain the directional characteristic that is an image of the source directional characteristic on a plane which is defined with two unchanged coordinates (Fig. 8).

3.1 The method of images for directional sources

Now, we have elements to define the method of images for directional sources placed in the corner of three perpendicular walls.

Let the planes of these walls be defined with axes of a global coordinate system (x=0, y=0 and z=0). The source position is at point $I_1(x_{01}, y_{01}, z_{01})$ of a global coordinate system, and also at the origin of a local coordinate system ($q_1=u_1=w_1=1$).

The total sound pressure in the region $x,y,z>0$ (p_t) can be calculated by summating the sound pressure of the source and seven image sources that are placed at points: $x_{0i}=q_i x_{01}$, $y_{0i}=u_i y_{01}$, $z_{0i}=w_i z_{01}$ (i=2,3..8). For source and his images we use the same relation for the sound pressure in their local coordinate system $p(x_i,y_i,z_i)$. Then:

$$p_t(x,y,z)=\sum_{i=1}^{8} p\left(q_i(x-x_{0i}),u_i(y-y_{0i}),w_i(z-z_{0i})\right) \qquad (21)$$

where the value of direction factors is given in Table 1.

i	1	2	3	4	5	6	7	8
q_i	1	-1	1	1	1	-1	-1	-1
u_i	1	1	-1	1	-1	-1	1	-1
w_i	1	1	1	-1	-1	1	-1	-1

Table 1. The value of direction factors

The proof is quite simple: to satisfy boundary conditions we need to prove that the normal component of the sound pressure gradient on rigid walls is equal to zero. If we apply the gradient operator on Eq. (21), we obtain:

$$\left(\partial p_t/\partial x\right)_{for x=0}=0 \quad \Rightarrow \quad \sum_i q_i = 0$$

$$\left(\partial p_t/\partial y\right)_{for y=0}=0 \quad \Rightarrow \quad \sum_i u_i = 0$$

$$\left(\partial p_t/\partial z\right)_{for z=0}=0 \quad \Rightarrow \quad \sum_i w_i = 0 \qquad (22)$$

that is, the sum of all direction factors must be equal to zero. Since the defined value of each direction factor can be +1 or -1, we have eight possible combination, shown in Table 1, to satisfy the boundary condition (22).

Eq. (21), giving the total sound pressure, can be further simplified to the form:

$$p_t(x,y,z) = \sum_{i=1}^{8} p\left((q_i x - x_{01}),(u_i y - y_{01}),(w_i z - z_{01})\right) \tag{23}$$

because the product of the direction factor and the appropriate image source coordinate is equal to the source coordinate ($x_{01}=q_i x_{0i}$, $y_{01}=u_i y_{0i}$ and $z_{01}=w_i z_{0i}$).

To calculate the total sound pressure using Eq. (23), we need the following data:

(1) the source position (x_{01}, y_{01}, z_{01}),
(2) values of direction factors (Table 1),
(3) an analytical expression for the sound pressure of the source in unbounded space.

If the loudspeaker directional characteristic is obtained by measuring the free-field response, then the analytical form of the directional characteristic has to be estimated from measured data by interpolation.

We've assumed that the loudspeaker axis is in z-axis direction. To estimate the response of a loudspeaker which is rotated for some angle α in the horizontal plane, we have to make the rotating transformation of a local coordinate system, that is, we substitute:

$$x \leftarrow x\cos\alpha - z\sin\alpha, \qquad z \leftarrow z\cos\alpha + x\sin\alpha, \qquad y \leftarrow y. \tag{24}$$

Similarly, if the loudspeaker rotates in the vertical plane for angle β, we substitute:

$$x \leftarrow x, \qquad y \leftarrow y\cos\beta + z\sin\beta, \qquad z \leftarrow -y\sin\beta + z\cos\beta. \tag{25}$$

Data obtained from the calculation of the frequency response can be used to estimate the loudspeaker in-room impulse response. We can use the Discrete Fourier Transform to calculate the impulse response, but to avoid time-aliasing, the frequency response has to be high-frequency limited. In a room acoustics analysis it is more appropriate to analyze the impulse response envelope e(i) (also referred to as Energy Time Curve). It can be obtained from a single-sided frequency response data using the Inverse Fourier Transform [7], that is:

$$e(t) = \left|\mathbf{F}^{-1}(Z(j\omega))\right| \tag{26}$$

where:
$$Z(j\omega) = p(j\omega) + \mathrm{sgn}(\omega)\cdot p(j\omega) = \begin{cases} 0, & \text{for } \omega < 0 \\ p(j\omega), & \text{for } \omega = 0 \\ 2p(j\omega), & \text{for } \omega > 0 \end{cases}. \tag{27}$$

In numerical calculations it is necessary to use the Discrete Fourier Transform to calculate Eq. (26) at equally spaced time points.

3.2 Examples of the loudspeaker frequency and impulse response

To demonstrate the method of images for directional sources, which are placed in the corner of three perpendicular walls, the results of the calculation of the frequency and impulse response for three types of acoustical sources will be presented. The sources are:

1. a closed, spherical box of volume 50 dm^3 (driver membrane diameter 20cm),
2. a planar, dipole loudspeaker (membrane dimension: 20x100cm),
3. a point nondirectional source.

Directional characteristics of a planar dipole loudspeaker are estimated by numerical calculation of the integral wave equation [7], and those for a spherical box are calculated using the solution of a wave equation presented in [3].

It is assumed that all sources have equal free-space response (shown dashed), frequency limited at 50Hz and 10500Hz (Butterworth 2. order response). The loudspeaker is located at: x_{01}=1.2m, y_{01}=1m, z_{01}=0.8m, and the loudspeaker axis is angled in the horizontal plane $\alpha = 30^0$ (referred to z-axis). The observer position is at: x=4m, y=1m, z=4m.

Figure 9. shows the frequency response curve (1/3 octave smoothed) for all source types, compared with a free-field response. The impulse response envelope is shown in Fig. 10.

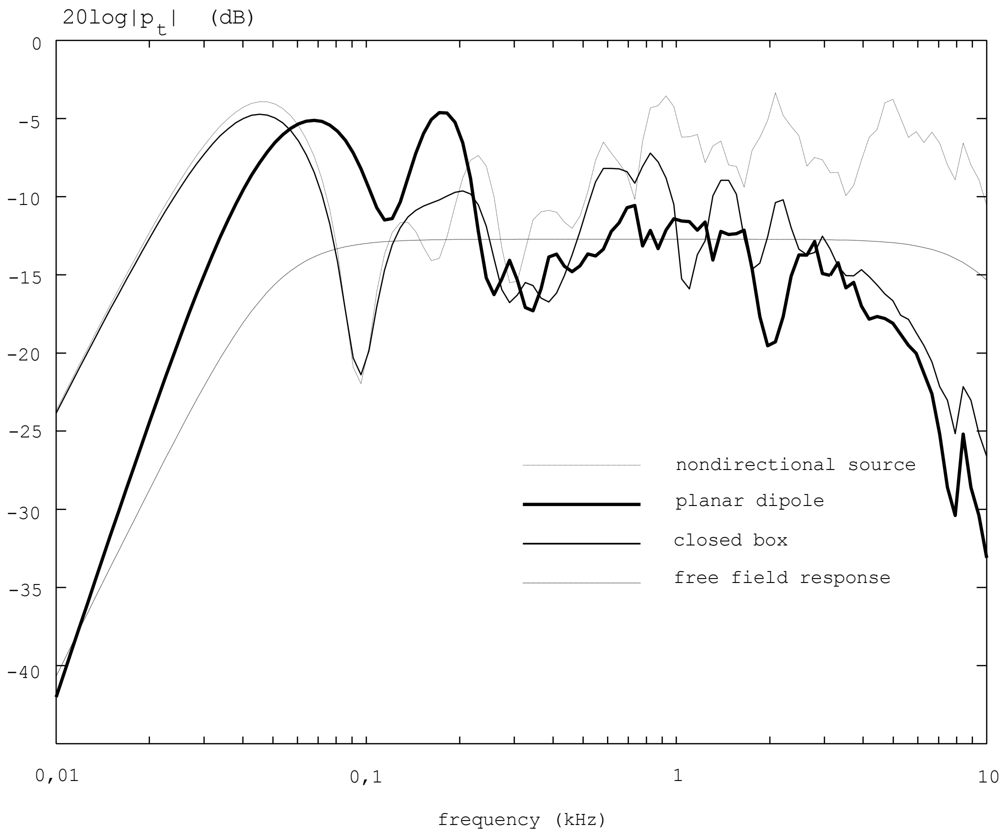

Fig. 9 The frequency response of a closed box, planar dipole and a nondirectional source that are placed in the corner of three rigid walls, compared with a free field response (x=4m, y=1m, z=4m, x_{01}=1.2m, y_{01}=1m, z_{01}=0.8m).

From these results some conclusions about the quality of the reproduced sound can be stressed. At very low frequencies the spherical box behaves as a nondirectional source, while the dipole loudspeaker has a reduced low frequency response. At higher frequencies directional sources have more even response; this is especially true for the planar dipole. The nondirectional source generates a high level of early reflections in the time interval 0-10ms. This can degrade the perceptual resolution and spatial properties of the reproduced sound image (due to the precedence effect). A much better quality of the reproduced sound can be expected from directional sources, especially from a planar loudspeaker, since it has low level of early reflections. These conclusions are presented as an example of the proposed method application in the estimation of psycho-acoustical properties of the reproduced sound.

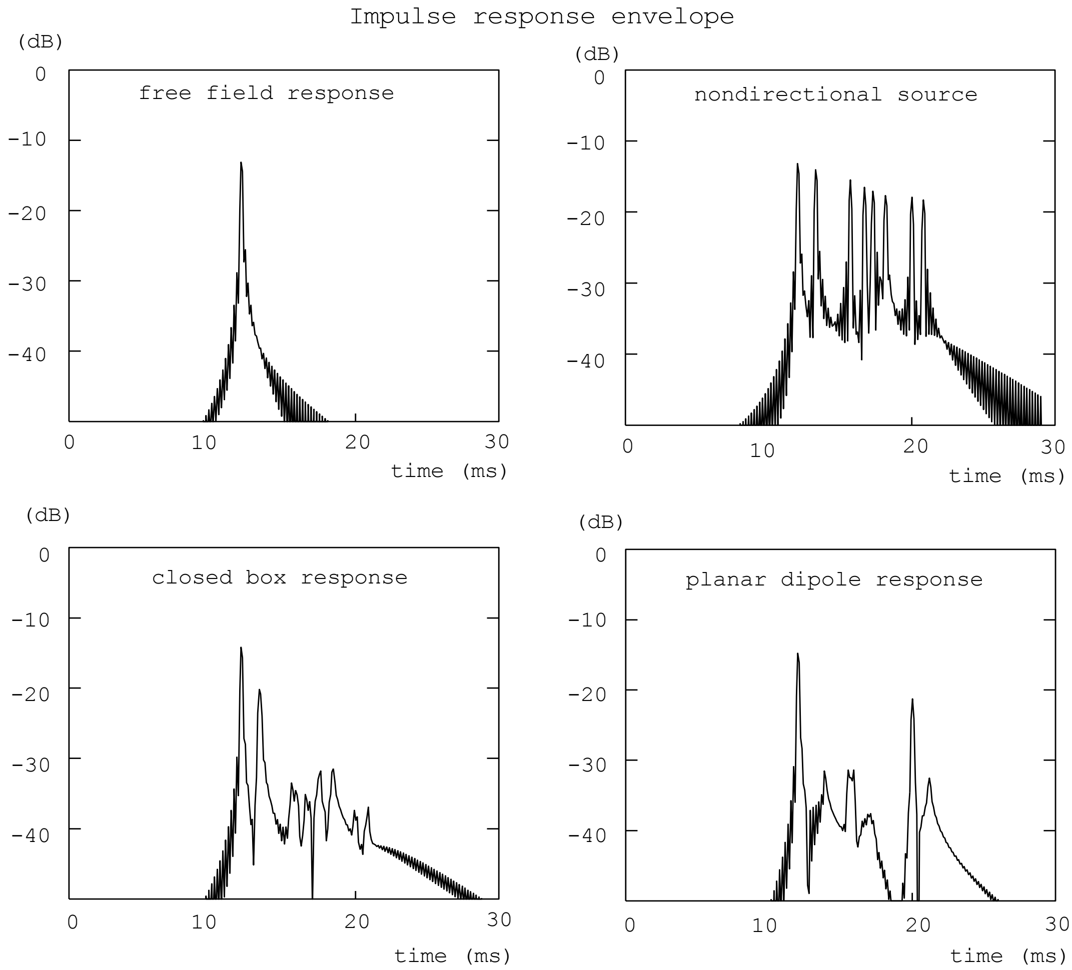

Fig. 10 The impulse response envelope of a closed box, planar dipole and a nondirectional source which are placed in the corner of three rigid walls, compared with a free field response (x=4m, y=1m, z=4m, x_{01}=1.2m, y_{01}=1m, z_{01}=0.8m).

4. CONCLUSION

If the nondirectional loudspeaker is mounted near reflecting walls, the method of images allows good estimation of the loudspeaker acoustic power response even in small rooms.

These methods fail to estimate the response of real loudspeakers at higher frequencies, where loudspeakers exhibit directional characteristics.

By approximating a loudspeaker box with a spherical baffle it has been shown that even small loudspeaker boxes exhibit directional characteristics on frequencies above 100 Hz. A simple solution for the $2\pi/4\pi$ bass response equalizer is derived.

The method of images for directional sources, which is presented in this paper, enables the analysis of the frequency and impulse response of real loudspeakers if they are located near reflecting walls. The use of this method is simple if the loudspeaker free field response is known.

5. REFERENCES

[1] A. D. Pierce, Acoustics, McGraw-Hill, New York, 1981.
[2] P. M. Morse, Theoretical Acoustics, McGraw-Hill, New York, 1968.
[3] E. Skudrzyk, The Foundations of Acoustics, Springer-Verlag, New York, 1971.
[4] R. F. Allison, The Influence of Room Boundaries on Loudspeaker Power Output, J. Audio Eng. Soc., Vol. 22, VII, 1974.
[5] K. O. Ballagh, Optimum Loudspeaker Placement Near Reflecting Planes, J. Audio Eng. Soc., Vol. 31, XII, 1983.
[6] G. Adams, Time Dependance of Loudspeaker Power Output in Small Rooms, J. Audio Eng. Soc., Vol. 37, IV, 1989.
[7] I. Mateljan, Acoustical Characteristics of the Wideband Loudspeaker Systems, Dr. sc. Dissertation, University of Zagreb, 1993.

Appendix - The Acoustic Power of a Point Source

The acoustic power P_A of a point source, which is located at $\mathbf{r_0}$, can be obtained by integrating the acoustic intensity on closed surface S, that is:

$$P_A = \mathrm{Re} \oiint_S \underline{p}^* \underline{\mathbf{v}} \cdot d\mathbf{S} = \frac{-1}{\omega\rho_0} \mathrm{Im} \oiint_S \underline{p}^* \nabla \underline{p} \cdot d\mathbf{S} . \qquad (28)$$

The radiation of a point source, with volume velocity U, at point $\mathbf{r}$ satisfies the nonhomogenous wave equation:

$$\nabla^2 \underline{p} + k^2 \underline{p} = -j\omega\rho_0 \underline{U} \delta(\mathbf{r} - \mathbf{r_0}), \qquad (29)$$

Multiplying this equation with p^*, we obtain

$$\nabla \cdot \left(\underline{p}^* \nabla \underline{p}\right) - \left|\nabla \underline{p}\right|^2 + k\left|\underline{p}\right|^2 = -j\omega\rho_0 \underline{U} \delta(\mathbf{r} - \mathbf{r_0}) \underline{p}^* . \qquad (30)$$

Integrating imaginary parts of this equation over the volume V, which is enclosed by surface S, and applying the Gauss theorem, we obtain the point source acoustic power:

$$P_A = \frac{-1}{\omega\rho_0} \mathrm{Im}\left(\oiint_S \underline{p}^* \nabla \underline{p} \cdot d\mathbf{S}\right) = \mathrm{Im}\left(\iiint_V j\underline{U} \delta(\mathbf{r} - \mathbf{r_{01}}) \underline{p}^* dV\right). \qquad (31)$$

The Dirac function reduces the volume integral on the right side of the Eq. (31) to the value of the integral function at point $\mathbf{r} \to \mathbf{r_0}$. Finally, the acoustic power of a point source is given by:

$$P_A = \mathrm{Im}\left(j\underline{U}\, \underline{p}^* (\mathbf{r} \to \mathbf{r_0})\right) = \mathrm{Re}\left(\underline{U}^*\, \underline{p}(\mathbf{r} \to \mathbf{r_0})\right) \qquad (32)$$

Appendix 2

For listener at position *x,y,z*, the distance of each image source is:

$$R_i = \sqrt{(x - q_i x_{01})^2 + (y - u_i y_{01})^2 + (z - w_i z_{01})^2}$$

If we need horizontal and vertical angle (FH, FV) at which sound reach the listener head

$$\varphi_V = \tan^{-1} \frac{y - u_i y_{01}}{z - w_i z_{01}},$$

$$\varphi_H = \tan^{-1} \frac{x - q_i x_{01}}{z - w_i z_{01}}$$

Delay from image sources referent to direct sound is:

$$D_i(\mathrm{sec}) = \frac{R_i - R_0}{c}$$